Triangular Sets in BPAS

1 TriangularSet Class Description

The `TriangularSet` class is a container class for handing triangular sets, *i.e.*, sets of polynomials of $\mathbb{K}[x_1, \ldots, x_n]$, with $\mathbb{K}$ a field, and variable ordering $x_1 < \cdots x_n$, with pairwise distinct main variables. This class is intended mainly to support the regular chain classes. For the notions of a regular chain and a triangular set see `https://en.wikipedia.org/wiki/Regular_chain` and `https://en.wikipedia.org/wiki/Wu%27s_method_of_characteristic_set#Triangular_set` and as the references therein.

The `TriangularSet` class consists of a variety of support functions needed to manipulate triangular sets, as well as routines designed to reduce polynomials with respect to a triangular set. In particular, the `normalForm` routine computes the normal form (in the sense of lexicographical Gröbner bases, see `https://en.wikipedia.org/wiki/Gr%C3%B6bner_basis`) of a polynomial with respect to a triangular set when the triangular set is strongly normalized (the initials of all of the polynomials are constant), and the `pseudoDivide` routine computes the pseudo-remainder of a polynomial with respect to the triangular set.

The most important basic feature of `TriangularSet` objects is that they come in two distinct types: fixed (intended for handling algebraic systems) and variable (intended for handling differential-algebraic systems). For fixed `TriangularSet` objects, the list of variables of the `TriangularSet` is fixed to be a certain ordered list of variables that cannot change over the life of the object. This is when the ambient space one is working in (explained below) is known and fixed. For variable `TriangularSet` objects, whenever a polynomial is added to the triangular set, any new variables in the polynomial are added to the list of variables of the `TriangularSet`. Both fixed and variable `TriangularSet` objects can be created with a list of transcendental variables, *i.e.*, variables that can never be subject to algebraic constraints. In both cases, the list of transcendental variables is fixed for the life of the object.

Objects of the `TriangularSet` class keep track of which variables v in the list of variables are subject to constraints, *i.e.*, for which there is a polynomial in the set with main variable v. Such variables are called *algebraic variables*, and are maintained in a special ordered list by the class (in the same order as the list of variables). To keep this distinction between the different kinds of variables clear, the list of variables are described in the documentation as "potentially algebraic", because they are the variables that can be subject to algebraic constraints. Thus, the intersection of the (potentially algebraic) variables and the transcendental variables is always empty, and the algebraic variables form a subset of the (potentially algebraic) variables. It is the (potentially algebraic) variables that determine the dimension of the ambient space of the triangular set, *i.e.*, the (affine) space over which the polynomials of the triangular

set act as constraints. The dimension of (the saturated ideal of) the triangular set is then the number of (potentially algebraic) variables minus the number of algebraic variables.